Literature Review on Anti-icing and Deicing Fluids Behavior Driven by Airflow

Pekka Koivisto, Aalto University
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<td>This literature review is aimed to increase the understanding of two phase flows with a thin liquid film – Newtonian or non - Newtonian – under an airflow. The purpose is to give basis for possible theoretical ad Hoc modeling of this kind of a fluid flow. The main objective in such a model would be to predict the flow off of de/anti-icing fluids from the surface of an aircraft wing during take-off.</td>
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<td>Erkki Soinne</td>
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FOREWORD

This report is focused on anti-icing and deicing fluids behaviour, which is an important subject in aviation safety. The report is part of the Icewing research conducted by the Finnish Transport Safety Agency Trafi in 2014. Due to a mishap the publication of the report was missed and is done first now.

Helsinki, 15.10.2019

Erkki Soinne
Chief Adviser, Aeronautics

Finnish Transport and Communications Agency Traficom
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1 Objectives and motivation of the review

This literature review is part of a Trafi funded project called “Icewing 3” which consists of both experimental and theoretical studies. One of the objectives of the project is to generate a theoretical “ad Hoc” – model for anti-icing fluid behavior and flow off properties on a flat plate subjected to an airflow simulating aircraft take-off situation. This literature review intents to assess the possibilities for such an ad Hoc model and to find a starting point and basis for the model.

The objective of this review and of the development of the ad Hoc-model is to produce practical results in terms of fluid volume flows when subjected to airflow instead of merely find out the details of two layer flow instability – as is the case in quite many of the studies referred in this study.

2 The scope of the review

The studies referred in this review are focused on aeronautical purposes in form of either anti-ice or deice fluid or water film behavior on wing surface or on a flat plate simulating the conditions on a wing. These studies are naturally of primary interests as the flow situation and values of the parameters such as air speed and fluid thickness are appropriate.

The present review is concentrated on theoretical modelling of two phase stratified flows though some experimental research supporting the evaluation of theoretical models are included in the reference list. The references in this study are reviewed in an approximate historical order. The aim of this is to give a correct impression of evolution of the subject.

3 Anti-icing, deicing fluid flows on flat plate and water film flows on flat plates or aircraft wing surface

3.1 Background

The beginning of scientific research on aerodynamic effects of anti/deicing fluids can be dated back to the late 1980’s and early 1990’s. In majority of these early studies the main objective was to establish a scientific basis for an aerodynamic acceptance test of the fluids. Most of the research included to this international research program involving several institutions and companies (AEA, SAE, Boeing, VKI, FAA, TC, etc) was experimental one. As a spin-off of the general interest on the phenomena however the first theoretical analyses focused on the particular problem of anti/deicing fluid appeared in 1990. Thereafter there was a steady flow of new studies throughout the 1990’s up to year 2002. After this only two studies have been published in 2010. Though the problem of this kind of two phase flow is far from solved the general interest for the problem seem to have reduced.

Part of the reviewed references are focused instead of anti/deicing fluids to the water film on wing surface. These studies are motivated either by studying the heavy rain effects on wing aerodynamics or the behavior of super cooled water on the wing surface during icing conditions.

3.2 Review of references

The particular methods addressing the problem of anti/deicing fluid or water film behavior on a wing may be divided roughly into four categories:

- linear (or nonlinear) stability analysis - mostly Orr Sommerfeld equations
- triple deck (or equivalent) theory of boundary layers
• methods ignoring the wave motion

• some kind of composition of different methods

As a starting point for Orr-Sommerfeld type of treatment of the problem is Yih’s study from 1990 (ref. 1). It is probably the most referred paper among the studies focusing particularly on anti/deicing fluids. Yih’s earlier paper from 1967 (ref. 2) is referred even more widely among all the stratified two phase flow studies not only those focusing on anti/deicing fluids. The kind of instability in two layer flow found in the first study (ref. 2) is nowadays called Yih’s instability and the wave motion described in the paper referred as Yih’s mode.

The problem setting in Yih’s study is simplified as follows:

• the 2 D wing section is reduced to a flat plate case

• the shear stress on the interface is calculated using Blasius flow

• for stability analysis both the air and liquid boundary layer velocity distribution is considered linear

• the variation of liquid thickness in direction of the airflow is ignored which implies a parallel stationary primary flow in the liquid

The formulation of the stability problem follows the general procedure for linear stability analysis:

• start with a laminar solution to the hydrodynamic equations (mass conservation and Navier-Stokes equations)

• perturb the laminar solution with infinitesimal disturbances sinusoidal in space and time

• substitute the disturbed solution into the hydrodynamic equations to derive the disturbance equations which leads to an eigenvalue problem for wave number and phase velocity of the disturbances. Instability is the studied based on the eigenvalue problem solutions

When infinitesimal disturbances lead to higher order terms which can be neglected in perturbation equation the stability analysis is considered as linear one.

In Yih’s analysis perturbation equations are constructed for both liquid and airflow using separate stream functions and applying boundary conditions for solid surface, interface of fluid and air and conditions outside the air boundary layer to describe the flows. Solution of eigenvalues leads to closed form algebraic equations for wave number and phase speed of the liquid waves. Along the solution of the eigenvalue problem a considerable amount of equations and connections between parameters are given referring to other publications – without explicit derivation in the study. The effect of gravity is included in Navier-Stokes equation however the effect of surface tension appears in equations in boundary condition for continuity of normal stress across the free surface.

The significant result considering the instability is – quoting ref. 1 – “instability under study arises only from the velocity gradients of the fluids at their interface, and that the exact form of the velocity profile of the air and in particular its curvature (to which fluid dynamicists have historically attached enormous importance), have little consequence here. The difference in the velocity gradients of the two fluids come from the difference in their viscosity, and therefore instability under examination here is of the kind found by Yih (ref. 2)”
The most interesting chapter in Yih’s paper is (Chapter 8) “Application of the theory to a special case”. In this chapter he compares the experimental data from Boeing wind tunnel tests for a 2D wing model (ref. 3) with the results given in his paper. The comparison appears to be quite inadequate without clear results to evaluate his theory.

The measurement case of ref. 3 which Yih is using as an evaluation case is a wind tunnel test for a single element wing section model with a chord of 0.28 m. The model was applied with Type II fluid and then subjected to an accelerating wind speed from idle to 60 m/s to simulate the take-off run of an airliner. The situation Yih is using for evaluation of his stationary flow situation is a snapshot from accelerating flow at point 5.6 s from start with a speed of 27.3 m/s. In particular he calculates some of the parameters (Re-number and shear stress) at a fixed chord wise position of 75% of chord from leading edge.

By calculating the number of waves from fig. 1 (copied from ref. 3 fig. 10) Yih receives the dimensionless wave number (referred to fluid initial thickness) of 0.5. To get a quasi-stationary viscosity value of the fluid for his computation he uses the graphical presentation of fig. 2. (ref. 3, fig.3). As the estimated shear stress is even below the lowest value in fig. 2. Yih use the value of 10 Pa*s, which is practically the static viscosity value of the fluid – being far away from the final shear thinning values.

Yih’s calculations gives for dimensionless wave number 0.33 and for interfacial velocity 0.0539 mm/s which in turn means practically a stagnant situation. Though Yih does not explicitly reveal his estimate to the wave phase speed as a result it is easily calculated from the given data to be 0.4 mm/s. One particular comment in ref. 1 on the instability mode is important – and it will be addressed later on this report when dealing with ref.19. According to Yih: “Although HH (=ref. 3) did not measure \( c_r \), their photographs taken at various times suggest that it is small, certainly less than the \( c_r \) predicted for the classical Tollmien Schlichting waves for Blasius flow, which is the order of 0.2. The \( \alpha^* \) for the most unstable mode in Tollmien Schlichting theory for Blasius flow is approximately 0.26, where \( \alpha^* \) is the wave number based on the momentum thickness of the boundary layer which is 0.534 mm for the case in hand. This \( \alpha^* \) corresponds to an \( \alpha \) based on fluid thickness \( d \) of only 0.126 far less than the experimental value of 0.5. Thus the waves photographed by HH (=ref. 3) do not seem to be classical Tollmien Schlichting waves”

Although Yih’s estimate on wave number is on the same playground (this will be corrected later on in ref. 19) with the experiment that he is using as an evaluation case, all the other values are obviously a total failure. It is easy to estimate from the two fluid profile curves of fig. 1 what is the volume transferred from the leading edge area to the trailing edge area through the selected point of 75 % of chord from leading edge – where the fluid thickness is luckily exactly the same 0.9 mm before and during the test (Yih’s theory presumed the thickness to be constant in chord wise direction). The time between these two situations is 5.6 seconds which gives an average fluid speed on the particular position of 215 mm/s. If assuming a linear speed distribution in the fluid (as Yih does) from bottom to surface the interfacial speed will be \( 2 \times 215 = 430 \) mm/s. It means 4 to 5 orders of magnitude more than Yih’s estimate. Regarding the wave speed author’s own experimental studies (ref. 4) with a 2D wing model gave wave speed of approximately 200 mm/s with a wind speed of 25 m/s so the Yih’s estimate of 0.4 mm/s is obviously not correct.

When trying to analyze what is wrong in Yih’s analysis the high value of viscosity (10 Pa*s) draws attention. This of course raises the question of shear stress used in Yih’s calculations. Though Yih uses the correct shear stress value for a flat plate it is seemingly low compared to the 2D wing shear stresses shown in fig. 2. These
in turn are exceptionally high values. Transferring the shear stress values of fig. 2 to friction coefficients a maximum value of near 0.1 is given at the leading edge area and a chord wise average value of 0.015 in the typical climb out attitude and at 133 kts. These values are extraordinarily high when compared to the flat plate values of fig. 3.

It actually seems obvious that according to figure 2 pure shear stress is never able to get the anti-icing fluid viscosity down to values at which it flows off – at least not during take – off run (the lower curve in fig. 2). This issue will be addressed further later on in this review.

In ref. 5, Salvador (1991) gives quite contradicting shear stress vs. viscosity values for the same Hoechst 1704 Type II anti-icing fluid that was used in experiments of ref. 3. The viscosity vs. shear stress figure of ref. 5 is presented in fig. 4. Comparing fig. 2 and fig. 4 the value of 5 Pa corresponds 0.1 psf which means the corresponding viscosities differs an order of magnitude between the two figures.

Considering now Yih’s results the fluid interface speed would have been in light of these viscosity values one order of magnitude larger than Yih reported. However there is still at least 3 orders of magnitudes to go when compared to experiments.

Nelson et al formulates a non-similar boundary layer theory for air blowing over water on a flat plate in ref. 6 (1995). Whereas in Yih’s study the instability of a known liquid depth profile (which in his case was constant) is considered, Nelson et al neglects the instability problem and finds the film thickness variation in an analytical form.

Nelson et al writes the dimensionless Navier Stokes equations – including continuity equation - for air and water (the properties of liquid are not restricted however the liquid is Newtonian fluid). Denoting x-coordinate as the direction of the flat plate and y as the perpendicular coordinate the boundary conditions are set as follows:

- velocity components in x and y directions are continuous across the interface (denoted as y=h(x))
- the shear stress is continuous across the interface
- the jump in the normal stress is balanced by interfacial tension (surface tension)

As the liquid surface position is originally unknown one additional equation comes out of the kinematic equation of the free surface.

Original equations are time dependent however the equations are approximated later on with equations where time as a parameter has disappeared without any notification. The approximation is asymptotic in the meaning that it holds once x (or its dimensionless form ξ is large enough)

Once the asymptotic solution for large x (or ξ ) is introduced it appears that it is based on a prescribed value of water flux (volume flow rate in the water) Q – which then will be constant in x-direction. This leads us to a situation where the only added value of this article is the shape of free surface that turns out to be proportional to $x^{1/4}$ ie.

$$h(x) = B x^{1/4} \quad (1)$$

where B is constant depending on prescribed Q. After the derivation of free surface in closed form the authors’ notice that the boundary layer of air (as well as water!) is a Blasius solution.
Figure 1. Fluid thickness profiles on the 2D wing model of ref. 3.
Figure 2. Chord wise shear stress distribution of the 2D wing model of ref. 3 and viscosity variation of anti-ice fluid with shear stress (psf = pounds per square feet)
What is left without consideration is that actually if the water flux is constant and the film thickness is increasing in $\frac{1}{4}$ power of distance from leading edge the mean velocity of water is reducing in $-\frac{1}{4}$ power of this distance. Experiments for a finite flat plate does not support this idea.

The wave motion on the surface is not addressed at all though the prevailing paradigm on the stratified two phase flow theories is that the waves play dominant role in mass transfer of the liquid. However this issue will be considered later on in connection of another report dealing with the case of Nelson et al.

Perron et al (ref. 7) introduces a real multidisciplinary study on de/anti-icing fluid flow off mechanism. As the phenomenon is not simple the authors tackle the problem from many directions using different methods to find out interrelationships between different parameters. The purpose of the work is said to be an introductory evaluation of shear stress and shear rate experienced at each stage of the history of the fluid: aircraft at rest, taxi and runway acceleration.
The study begins with an introduction to basic rheology essential in analysis of non-Newtonian anti-icing fluids. The routine rheological measurements within industry are done with Brookfield Viscometers with certain standard spindles. However the quality assurance values laid down in SAE standards refer simply to Viscometer spindle rotational speeds. To convert spindle speeds to shear rates of the anti-icing fluids is not possible directly by conversion factors. A viscometer is calibrated to give a viscosity value for a Newtonian fluid at the surface of a spindle with certain rotational speed and measured torque. This value – called apparent viscosity – is used as a first estimate to calculate the power law of viscosity vs. shear rate applying curve fitting for viscosity vs. shear rate relationship to find the power $n$ in equation:

$$\mu = k \dot{\gamma}^{n-1} \quad \text{where} \quad k = \text{constant} \quad \text{and} \quad \dot{\gamma} = \text{shear rate [1/s]} \quad (2)$$

In this first curve fitting the shear rate is approximated from a Newtonian relationship between rotational speed and shear rate at spindle surface. The power law connection in turn leads to a new estimate for a shear rate of non-Newtonian fluid from an equation including the exponent $n$ in the power law (2) of viscosity. Calculation of non-Newtonian viscosity leads to a recursive scheme since the exponent $n$ in the power law is initially unknown. The correction to viscosity vs. shear rate is due to the pseudo plastic behavior of non-Newtonian fluid some 20 to 50% less than "apparent viscosity" – see figure 5 from ref. 7. According to ref. 7 the exponent is approximately $n = 0.5$ for most of the Type II fluids (at that time 1995).
Once applied on the wing anti-icing fluid is subject to thinning out while draining off the inclined surfaces of the leading edge area. When the Hold Over Time (HOT) of a fluid is established two tests (Water Spray Endurance Test = WSET and High Humidity Endurance Test) are conducted. In these tests 10 cm times 30 cm flat aluminum plates are applied with applicant fluid which after they are positioned to a 10° inclination angle to simulate the draining process of the wing upper surface on the leading edge area. After 5 minutes time the when the fluid thickness has stabilized the precipitations (freezing fog in WSET and frost in HHET) are turned on with predetermined mass flow. The time when ice, forming from the top of the plate has reached 25 mm length is recorded as HOT.

This process described above is the topic of the next chapter of ref. 7 after basic rheology. There is a mathematical model laid down for the draining process. The flow is assumed as a “creeping” one so the convective terms in Navier Stokes equations are neglected and pressure distribution is considered hydrostatic. A 2D model is created and the numerical solution is found using the PHOENICS CFD-package (note year 1995). There is a figure of numerical domain in figure 6 (from ref. 7).

The draining results of the CFD – model in ref. 7 are presented for some reason for constant viscosities (Newtonian). In figure 7 shear rates are shown in different fluid depths (vertical axis called height in µm) with different viscosities at the position of 25 mm from the top of the plate and after 5 min from start of the test. As shear rates are linear in fluid depth direction the velocity distribution is
Parameter $u$ in fig. 7 is wind speed outside fluid layer. As wave motion is not considered the maximum wind speed is 5 m/s to avoid the wave motion. The effect of external air velocity is said to be “sensitive” only at moderate viscosity (fig. 7)

Figure 7. Shear rate at position $x=25$ mm and $t = 5$ min. (ref. 7)

Maximum shear rates, which appear at the bottom of the fluid layer, are shown in figures 8 and 9. High values of shear are found within first five minutes which motivates the 5 min initial settling time before the test.

Figure 8. Maximum shear rate at $x = 25$ mm (at the bottom of the fluid)
The next chapter of ref. 7 is called “Fluid entrainment On-set”. This concept is not an established one but just used in this particular article. In ref. 7 the velocity of entrainment on-set $V_{eo}$ means the (minimum) air velocity required for the wave generation at the air-fluid interface. According to ref. 7 before the wave generation the fluid thickness (on the wing or flat plate) and the fluid viscosity (of thickened non-Newtonian fluid) do not vary significantly, and the viscous behavior is close to Newtonian. But after the wave generation, the shear stress and the shear rate are very high and the pseudoplastic (shear thinning) behavior appears. The particular interest regarding the behavior of the air/fluid interface is to determine if there is significant fluid entrainment during taxi, as well as to identify the initial stage of fluid movement during ground acceleration (take-off roll).

The following stability analysis description in ref. 7 is far from being explicit and clear. The calculation method is said to be the one in Yih’s study (ref. 2). After introducing Yih’s equation for complex phase velocity (real part = phase velocity, imaginary part = growth rate/wavenumber) there is a figure (figure 10) shown as for a “typical” growth rate.

Figure 9. Maximum shear rate at $t = 5$ min.

Figure 10. Growth rate in function of wavenumber
This is followed by a statement that "wave formation can be considered to occur at the maximum growth rate". Then it is speculated that "since (Yih’s) equations are laminar, the position of wave generation is assumed to be located at the laminar/turbulent transition" – see figure 11.

Figure 11. Waves generation position

Under the next chapter title of “Calculation of $V_{eo}$” there is definitely no word on how this speed is calculated – however there is a fair sensitivity analysis of parameters that do or don’t effect on $V_{eo}$. In Table 1 there is the reference values for the fluid and in table 2 the ranges of parameters varied in sensitivity analysis. Finally the results of this analysis is shown on figure 12.

Table 1. Reference fluid for sensitivity analysis of $V_{eo}$

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
<tr>
<td>Density ($\rho$)</td>
<td>1.05 kg/m$^3$</td>
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<tr>
<td>Surface tension ($\Gamma$)</td>
<td>35 dynes/cm</td>
</tr>
<tr>
<td>Thickness ($h$)</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Air Temperature</td>
<td>0 °C</td>
</tr>
<tr>
<td>Viscosity function ($\mu$)</td>
<td>$\mu = 2.82 \cdot \gamma^{0.46}$ Pa·s</td>
</tr>
<tr>
<td>Transition Reynolds Number</td>
<td>$5 \times 10^5$</td>
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Table 2. Variation range for parameters in sensitivity analysis

<table>
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<th>Parameter</th>
<th>Range</th>
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<tr>
<td>Density</td>
<td>$1.0 \leq \rho \leq 1.2$ (kg/m³)</td>
</tr>
<tr>
<td>Surface tension</td>
<td>$30 \leq \Gamma \leq 40$ (dynes/cm)</td>
</tr>
<tr>
<td>Fluid thickness</td>
<td>$1.2 \leq h \leq 1.8$ (mm)</td>
</tr>
<tr>
<td>Air Temperature</td>
<td>$-23.2 \leq T_{air} \leq 6.9$ (°C)</td>
</tr>
<tr>
<td>Transition Reynolds Number</td>
<td>$10^5 \leq Re_{V_{eo}} \leq 10^6$</td>
</tr>
</tbody>
</table>

Figure 12. $V_{eo}$ variation with fluid parameters.

Interesting consequences of this sensitivity analysis are:

- surface tension and fluid thickness have negligible effects
- though the growth rate of the perturbation depends on the viscosity the velocity value which initiates the perturbation does not (this has not been explicitly deduced in the article !)
- the influence of fluid density is not negligible but modest

There is a short notice of “$V_{eo}$ value stays above 14 m/s which represents a fairly high value for taxi”. Firstly: there is no comment on how this value is calculated and secondly it is not a high value but a very average value for taxi speed especially when the head wind values are accounted for. The $V_{eo}$ value of 14 m/s is quite well in line with authors own measurements (ref. 4)

The last two figures related to this sensitivity analysis are figures 13 and 14 below. The first one shows the dependency of $V_{eo}$ on air temperature (Fig 13.) and the second one is $V_{eo}$ dependency on transition Reynolds number figure 14 is
a real mystery as there is no words about how and with what method $V_{eo}$ has been estimated as a function of transition of boundary layer.

Figure 13. $V_{eo}$ dependency on air temperature.

Figure 14. $V_{eo}$ variation with Reynolds number of transition of air flow.

The next chapter in ref. 7 is titled "Fluid take-off shearing". The purpose of the chapter is said to determine the fluid viscosity, shear stress and shear rate as a function of air velocity during ground acceleration. As it is admitted that it is "not yet possible" to predict numerically the evolution of both media (fluid and air) there is an inverse technique selected to determine the shear stress on the surface. To begin with the boundary layer displacement thickness is taken from a standard fluid Aerodynamic Acceptance Test (as per ref. 8) during the 30 s acceleration phase (see fig. 15). This experimental result is then used to find a
flat plate boundary layer with an equivalent sand grain roughness using an integral method based on Coles’ law of wake. As the equivalent sand grain thickness has been found to correlate the

Figure 15. A boundary layer displacement thickness test (BLDT) of an anti-ice fluid.

measured displacement thickness the corresponding boundary layer friction coefficient and shear stress may be calculated. The next step of reasoning in ref. 7 is to expect the velocity profile to be close to linear and, therefore, the stress in the fluid is estimated to equal with the interface shear stress. For some reason the wave motion and its effects on shear stress have been rejected. Some comments and speculation on this are presented later on this report considering another publication of authors of ref. 7.

Figure 16 presents the calculated shear stress on the surface of fluid as a function of air velocity for four different fluid. When transferred to friction coefficient \( C_f \) the three non-Newtonian fluids (three highest curves) appear to give values of \( C_f = 0.011-0.015 \). The Newtonian (MIL) fluid give a value of roughly half of this.

Using the rheological data presented earlier in ref. 7 the shear rate and the viscosity variation with air velocity are presented in Figures 17 and 18.
Figure 16. Shear stress variation with air velocity.

Figure 17. Shear rate variation with air velocity. Note the poor quality of the original figure. As MIL-fluid is the only Newtonian fluid the highest curve is probably for MIL-fluid.

Figure 18. Viscosity variation with air velocity.
The shear rate is commented in ref. 7 to be twice as large as the one experienced at the maximum rotation velocity of the viscometer (60 rpm). According to ref. 9 the viscometer shear rate is roughly 4nN/n, where N= rotation velocity (s⁻¹) and n= exponent of the power law of the fluid (2). Using the maximum N and n = 0.5 one gets maximum shear rate of around 25 s⁻¹. Also the shear rate is commented to be quite linearly varying with air speed (fig. 17). The viscosity “break down” due to high shear is said to be complete typically before 40 m/s.

In conclusion of ref. 7 there is the following statement: “…complete numerical prediction (for the take-off stage) will require coupling of the wave generation mechanism with the rough turbulent air boundary layer”. This is easy to agree with.

In ref. 10 (1996) the authors of ref. 7 review their study once again with some additions to ref. 7. The first addition is what they call as “validation” of their boundary layer calculation method which is performed by comparing their results for C_f with the well-known Schlichting formula for a rough flat plate:

$$C_f = (2.87 + 1.58 \lg(x/k_s))^{-2.5} \quad (3)$$

where x = distance from leading edge and k_s = height of roughness element. It should be noted that Schlichting formula does not apply for grain size larger than

$$k_s = x/100$$

This will limit the C_f below a value of approximately 0.0155. See also fig. 3. It is obvious that from some point on the pure shear stress and friction coefficient does not explain any more the flat plate drag as grain size is increasing but there will be also pressure related forces involved.

The comparison between ref. 7 and 8 integral method and Schlichting formula for C_f is shown in Figure 19.

![Figure 19. Friction coefficient calculated by method of ref. 7 and Schlichtings formula. Here y* is friction height which may be defined by equation \(y^2 = \rho_0 u_a^2/\tau_a\), where \(\rho_0\) = density of air and \(u_a\) = kinematic viscosity of air.

The other new aspect was to introduce a so called roughness density parameter \(\Lambda\) which took into account also the roughness density. This parameter were derived from image analysis of the waves by calculating the ratio of the total surface area to the roughness area measured at the mid height of the wave. In this way the roughness height \(k_s\) is converted to an equivalent \(k_g\) which has the same spacing
as waves. Results of this analysis are listed in table 3. Figure 20 defines some of the parameters in table 3.

![Figure 20. Definition of wave geometry parameters in Table 3.](image)

Table 3. Experimental and numerical data for three BLDT-tests (two points of each test)

<table>
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<tr>
<th></th>
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</tr>
<tr>
<td>μ, cP</td>
<td>110</td>
<td>280</td>
<td>980</td>
</tr>
<tr>
<td>τ, Pa</td>
<td>3.1</td>
<td>5.0</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Re_x: 0.94 $10^6$ 3.29 $10^6$ 1.36 $10^6$ 3.33 $10^6$ 1.31 $10^6$ 3.32 $10^6$

C_f: 0.0147 0.0041 0.0113 0.0078 0.0149 0.00942

The following comment on the results of Table 3 is a direct quote from ref. 10:

"Comparison of actual fluid roughness with equivalent solid roughness can be done on the basis of the elements interspacing ($\lambda$ vs $\lambda_s$) and elements height ($d'$ vs $k_e$). For the two situations examined here, the order of magnitude is equivalent for solid and fluid roughness. Similar parameters are in agreement within about 10 %, with two exceptions for interspacing and one for height. It is therefore fair to assume that the movement of the wave does not have a specific influence on
the air boundary layer since fluid interface has the same effect as a solid roughness.” Note: the text is unedited quote.

The last reasoning on irrelevancy of wave motion sounds somewhat incomplete as the measured and calculated boundary layer have been in the first place tied together with the experimental result of displacement thickness.

The author of present report has added the values of Re_x and C_f below the table 3. The viscosity values in table 3 are based on the assumption of linear velocity profile in the fluid as it implies a constant shear stress across the fluid layer. Neglecting the surface tension the tangential force equilibrium requires at the interface:

\[ \tau_a = \tau_f \]

where subscript a denotes for air and f for fluid. According to reasoning of ref. 10 it then suffices to calculate the shear stress in the air layer, at the air/fluid interface to obtain the fluid shearing stress and the viscosity of a shear thinning fluid.

In the concluding remarks of ref. 10 there is a contradictory remark on wave entrainment onset velocity compared to ref. 7 of the same authors. Whereas \( V_{eo} \) is said to be approximately 14 m/s it is now reported that first waves appear at the velocity of 8 m/s. The fluids under consideration are the same on both studies.

In conclusions of ref. 10 the new “paradigm” of shearing forces are stated as follows: “The roughness analysis using an integration of the rough turbulent boundary layer demonstrates that the air/fluid interface effect is only geometrical and the movement of the wave does not have a significant effect on boundary layer development. Thus, the boundary layer development over a fluid film can be simulated by a solid roughness boundary layer on a flat plate with the same height and spacing as those of waves. However, at this time, the wave analysis is not sufficient since it does not provide the height of the wave (only interspacing).”

In the last sentences the authors admit that: “The method developed gives a better understanding … but it is not complete since it does not permit the simulation of the boundary layer development without the help of experimental data.”

In ref. 11 Boelens et al. (1996) presents what they call a “simulation” of flow over an airfoil with a thin layer of liquid. The paper is divided in two parts: first an introduction of a numerical method of calculating time dependent boundary layer on flat plate with a surface roughness, and second a linear hydrodynamic stability analysis of a non-Newtonian power law fluid sheared by gas flow (on a flat plate).

The first part includes a derivation of non-dimensional Prandtl boundary layer equations in a stream function formulation. The boundary layer model has a roughness element feature incorporated and a semi empirical transition model. Turbulence model for turbulent boundary layer is said to be the Cebeci-Smith model (algebraic turbulence model). After this the author presents results for a flat plate of 0.5 m long with and without roughness of 0.5 mm. A comparison with classical boundary layer formulas are done with good agreement. Just one question remains: why all this effort when all this data can be gathered from basic textbooks of fluid dynamics. Two last sentences in the “Conclusions” chapter of the first part is revealing: “...fluid surface roughens the surface of the airfoil due to the formation of “instationary” waves. As the shape of the waves is not exactly known, it is not possible to calculate the effect of these waves on the boundary layer flow”.

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In the second part of the paper an Orr-Sommerfeld equation for non-Newtonian power law fluid is derived from the same basic assumptions (linear small perturbation and nearly parallel flow) used in Yih’s paper with the exception that the stress tensor in Navier Stokes equation is now a non-linear function of rate of strain tensor as in Yih’s paper the viscosity was considered as “point vise constant”. Using the same assumptions for velocity distributions (Blasius profile for air and linear profile for fluid) the authors of ref. 11 end up to same kind of differential equations for stream functions of air and fluid however with the difference that the fluid equation now includes the constant \( n \) which is the exponent of the non-Newtonian power law of the fluid. Boundary conditions at the surface of fluid are the same as in Yih’s case:

- continuity of tangential velocity
- continuity of normal velocity
- continuity of tangential stress
- continuity of normal stress

The last condition brings the surface tension into the solution. There is also applied a boundary condition at infinity where all perturbation velocity approach zero. After adopting a concept called virtual interface where the air velocity (Blasius) profile has a “nearly” constant x-component and a small y-component the authors of ref 11 end up to a much simplified Orr Sommerfeld equations.

In the “Results” chapter there is comments on convergence of the numerical scheme authors have used and on unstable modes observed: “In the calculations two unstable modes were observed. The first mode is so called Blasius mode as it coincides with the curve (probably the \( c_i - \alpha -\text{curve} \), where \( c_i \) is the imaginary part of complex wave speed (negative for stable case) and \( \alpha \) = non dimensional wave number) of pure Blasius flow. This mode is stable for Re-numbers below the critical Re-number for Blasius boundary layer flow (!!). The second mode is called the interfacial mode. As this mode is absent in the Blasius boundary layer flow it is concluded that this mode originates from the presence of the liquid layer (!!!). For all Re-numbers observed this mode has always unstable wave-numbers (!!!!). The results of ref. 11 give quite modest added value for the understanding of anti-icing fluid flow under airflow.

A quite different approach from the previous studies is ref. 12, by Tsao 1996) et al. A high Re-number study for small disturbances is applied with asymptotic expansions for an air flow over a two dimensional liquid coated airfoil. The surface is not limited to a flat plate but there is a parabolic leading edge geometry considered as well. The liquid is limited to be a Newtonian one. The overall flow situation and the leading edge flow is described in figure 21 a) and b).

Ref. 12 starts with a derivation of dimensionless (global) boundary layer equations out of Navier Stokes equations using asymptotic expansions. The liquid layer is assumed to be the plane Couette flow (linear velocity distribution once again). The kinematic interface condition together with volumetric conservation leads to wave equations form liquid film. These equations are numerically solved for film flow over parabola aligned with the air flow in figure 22.
Figure 21. a) The global airfoil flow, b) leading edge flow, c) triple deck, d) lower deck

The solution of fig. 22 is commented in ref. 12 as follows: “Since this result appears to be in contradiction to experimental observations, both in wind tunnel and flight tests which do show significant wave development, more localized structures, including surface tension, gravity and pressure gradient were sought.”

It turned out in ref. 12 that the so called triple – deck theory addresses the local structure problem in a manner the global boundary layer structure is not able to do. The triple-deck theory was originally derived by Stewartson (ref. 13, 1969) to solve the flat plate trailing edge flow related problems the global boundary layer theory could not address. There is a schematic Figure on triple-deck configuration related to present problem in fig. 21 c) and d).
The following is a direct quote from ref. 12:

"As shown in fig. 21 the stream wise length of the triple-deck which is centered at an arbitrary position $s$ is $L \varepsilon^3$, where $L$ = reference length and $\varepsilon = \text{Re}^{-1/8}$ using the notation of ref. 12. The triple deck consists of three distinct vertical regions. The middle region (so called main deck) where $y = n/L \varepsilon^4$ and $X = s/L \varepsilon^3$ are the order one ($O(1)$) coordinates ($n$ = dimensional surface normal coordinate and $s$ = dimensional surface coordinate) is basically a displaced boundary layer flow, inviscid but rotational in nature. Its main function is to passively communicate the pressure displacement interaction between the upper and lower decks. Next is the outer region (outer deck), where $y' = n/L \varepsilon^3$ and $X$ are the relevant coordinates. The flow in outer region is inviscid and irrotational. The governing equation for the outer deck is the elliptic Laplace equation. The lowest region (lower deck), where $X$ and $Z = n/L \varepsilon^5$ are the order one coordinates, is a viscous sublayer which is needed to satisfy no-slip conditions on the interface. The governing equations obtained, to leading order in $\varepsilon$, are the conventional boundary layer equations subject to an outer boundary matching condition involving an interaction law which relates the viscous displacement effect and the inviscid outer deck pressure gradient according to localized thin airfoil theory. The full consistency arguments and asymptotic expansions in the three necessary regions are the same in their essential points as those laid down originally by Neiland (ref. 14), Stewartson and Williams (ref. 13) and Messiter (ref. 15). Details will not be given here."
As seen in figure 21 d) the liquid layer is beneath the lower deck. Because of the thinness of the liquid layer and the large value of viscosity ratio of liquid and air, the fundamental problem in the liquid layer reduces to Poiseuille flow that balances the viscous effects, induced pressure gradient, gravity and surface tension.

Substitution of new triple deck expansions into the Navier Stokes equations in conjunction with the boundary conditions – to first order – gives the standard triple deck equations for the lower deck (the variables shown are Prandtl transposed variables, see Stewartson (ref. 13)). The liquid layer is governed by Poiseuille equations.

The derived triple deck equations are not solved in any manner – instead it is stated that the triple deck structure shows a strong dependence on the location where it is positioned along the airfoil surface because the triple deck equations cannot be normalized with respect of various constants appearing in the equations such as the inviscid surface velocity and the film thickness whereas it is a well-known fact that the classical transformed triple deck problem, with a proper rescaling is free from such dependence (as the trailing edge flow ?).

Instead of a general solution there is (once again) a stability analysis presented. The variables are expanded to include the periodic disturbance term (as usual) and the perturbed variables are substituted to the governing equations to get the perturbation equation. What follows is a complicated set of calculation ending up to the following statement on conclusions:

1) as the initial liquid film height tends to zero asymptotically within the linear regime of triple deck structure, the most linearly unstable mode starts shifting toward the high frequency, short wave length bands with diminishing growth rate.

2) as the initial film height tends to infinity the most linearly unstable wave grows rapidly with constant limiting frequency provided both the surface tension factor \( \text{Re}^{-13/8} (\rho \sigma L/\mu^2) \), where \( \sigma \) is surface tension between liquid and air and all the other material values refer to air, \( L = \) reference length) and gravity factor \( (= Fr^{-1}) \) are small or moderate. If the surface tension and gravity factors were large the film flow would be stable.

These statements are shown in form of curves in figures 23, 24 and 25.
Figure 24. Critical frequencies \((k_1, k^*, k_2)\) versus dimensionless initial film height (see text of Figure 23). Critical frequencies are defined in Figure 25.

There is preliminary applications of the theory presented in the end of ref. 12. Two of them are of special interest. In the first one the theory of Nelson et al. (ref. 6) presented earlier on this report with a liquid layer on flat plate driven by wind is considered. In ref. 6 the instability was neglected and the smooth liquid surface was of a form of \(h(x) = Bx^{1/4}\). Tsao et al. does now a stability analysis applying their triple deck theory and get a result in form of figure 26 where \(x_c\) is the distance from leading edge where the first waves appear on the liquid surface given by the equation above for different wind speeds and initial liquid depths. The \(x_c\) coordinate is dimensionless and there no hint how it is made dimensionless. If the reference is the flat plate length \(L\), all the waves seem to appear so near the leading edge that the differences is without practical interest.
Figure 26. The points of loss of stability for a water film on a flat plate as per ref. 6.

The second application of the theory of ref. 12 is even more interesting as it is the case in Yih’s paper (ref. 1) described earlier in the present report. Yih compares the instability of anti-icing fluid experiments on a wing section to his flat plate calculations. Now Tsao et al. compares the results of their theory for an equal flat plate case with Yih’s results. The parameters compared are the dimensionless wave number ($\alpha$), growth number ($\sigma$) and phase velocity ($c_r$). The comparison is concluded to the table below.

<table>
<thead>
<tr>
<th>Author</th>
<th>Stable flow</th>
<th>Most unstable flow</th>
<th>Growth rate</th>
<th>Phase velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yih (ref. 1)</td>
<td>$\alpha &lt; 0.1$ \quad $\alpha &gt; 0.5$</td>
<td>$\alpha = 0.33$</td>
<td>$\sigma = 5 \times 10^{-6}$</td>
<td>$c_r = 1.46 \times 10^{-5}$</td>
</tr>
<tr>
<td>Tsao et al. (ref. 12)</td>
<td>$\alpha &lt; 0.009$ \quad $\alpha &gt; 1.326$</td>
<td>$\alpha = 0.893$</td>
<td>$\sigma = 3.67 \times 10^{-5}$</td>
<td>$c_r = 5.467 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

When we recall the experimental values of ref. 3 and 4 it is easy to note that there is at least no improvement in results compared to Yih’s analysis. Though the phase speed is almost 4 times larger than in the case of Yih it means only a dimensional speed of 1.5 mm/s which is still two orders of magnitude smaller than measured ones (ref. 4). The study of ref. 12 is practically duplicated word by word in ref. 16.

Myers and Thompson (ref. 17) introduces a three dimensional mathematical model for a flow of thin film of water driven by air shear. They too neglect the
stability issue of the flow as Nelson et al. in ref. 6. The differences to the analysis of Nelson et al. are:

- Flow is basically 3D (some results in 2D given below) and the solid body surface (called substrate in the article) may be at an angle other than perpendicular to the gravity vector. The aerodynamic forces are included to a pressure term $p_a$

- Liquid layer is assumed to be so thin that the so called lubrication model is valid. In essence this means that convection terms in momentum equation have been neglected (as in Stokes flow)

- Air boundary layer above the liquid has been replaced by a constant shear stress ($T$). This naturally simplifies the mathematics of the problem significantly

Similarities between Myers and Thompson and Nelson et al. are:

- Gravity and surface tension are included

- Boundary conditions including the kinematic condition of free surface are similar

- Water is injected on the surface at a constant flux $Q$

There is a very interesting special case calculated in ref. 17 which gives a possibility to compare the results of Nelson et al. and Myers and Thompson methods. This case is a 2D flat plate at a 45° angle to gravity vector however with a zero pressure gradient in the axis of the plate which means a parallel airflow. The only force that is not in line with the case of Nelson et al is now the gravity that has a component parallel with the air flow. In Myer and Thompson’s example the following are known parameters:

- Initial film thickness at the leading edge ($x=0$) is 2 mm

- Water is injected $10^{-4} \text{ m}^2/\text{s}$ per unit length (in the third dimension!). This implies a 0.05 m/s average initial velocity on the leading edge

- There are three different shear stress values representing the airflow as parameters: $T_1 = 0$, $T_2 = 1 \text{ Pa}$, and $T_3 = 10 \text{ Pa}$. These values are said to be equivalent of air speeds of the order of 0, 100 and 1000 kph (?) which probably means km/h so the corresponding values in m/s would be: 0 m/s, 28 m/s and 278 m/s. To assess how realistic these values are the corresponding friction coefficients would be $C_f = 0$, 0.0021 and 0.00021. On the other hand a speed of 278 m/s would imply some compressibility effects too (!).

The results of this example are shown in figures 27 and 28.

Myers and Thompson derives also a closed form for limiting fluid thickness for maximum shear (gravity may be neglected). This formula is:

$$h_s = \left(\frac{2\mu Q}{T}\right)^{1/2}$$

It is an interesting detail that the constant $B$ in the equation for liquid film height equation (1) as per Nelson et al. may be written as:

$$B = \left(\frac{2Q}{A}\right)^{1/2}$$ where $A = \mu_2 k f''(0)/\mu_1$

Here $\mu_2$ is viscosity of liquid, $\mu_1$ the viscosity of air and $f''(0)$ is the second derivative of Blasius function $f(\eta)$ at the solid surface. As it is generally known the shear stress in Blasius boundary layer is proportional to $\mu_1 f''(\eta)$. This implies that the proportionalities between liquid height and liquid viscosity, liquid flux and air generated shear are exactly identical between the Nelson et al and Myers and Thompson reports.
Figure 27. Liquid film shape for varying air shear stress $T = 0, 1$ and $10$ Pa. Limiting values are $h_g$ (no shear only gravity) and $h_s$ (maximum shear of $10$ Pa).

Figure 28. Average liquid velocity for varying air shear stress. Limiting values $u_g$ (gravity only) and $u_s$ (maximum shear of $10$ Pa)
However the liquid heights with distance from leading edge differs essentially. So does the mean velocities: as Nelson et al receives a decreasing velocity with distance Myers and Thompson end up to a limiting constant velocity.

All the previous stability analyses have been based on parallel or nearly parallel flow assumption and have so been local temporal stability analysis for the problem of waves in liquid. Boelens and Sijp (ref. 18, 1998) study the instability problem for non-parallel primary (undisturbed) flow and in this way they investigate also the spatial evolution of an instability. The study is focused on the familiar flat plate case with a thin film of liquid on the surface. Flow is 2 dimensional wit x – coordinate parallel to the plate and y- coordinate perpendicular to it with the origin 0.25 m ahead of leading edge of the plate.

Decomposing the flow field into the primary (normally laminar) flow and a perturbation (secondary flow) we get:

$$v(x,y,t) = V(x,y,t) + v'(x,y,t)$$
$$p(x,y,t) = P(x,y,t) + p'(x,y,t)$$

where the capital variables represent the primary flow and the dotted variables the disturbances (normally small). In the stability analysis there is often a stream function $\psi$ adopted to substitute the continuity equation. Then the velocity components are partial derivatives of the stream function. In the conventional local (temporal) stability analysis where the primary flow is considered parallel the secondary flow components $\psi'$ and $p'$ (disturbances) are decomposed as follows:

$$\psi'(x,y,t) = \phi(y) e^{i(\alpha x - \omega t)}$$
$$p'(x,y,t) = f(y) e^{i(\alpha x - \omega t)}$$

In the parallel flow type of analysis the amplitude functions $\phi(y)$ and $f(y)$ depend only on the coordinate normal to the surface, $\alpha$ is the complex wave number and $\omega$ is the complex frequency, both independent of $x$ and $t$.

To include the effect of non-parallel primary flow we decompose the secondary flow components as follows:

$$\psi'(x,y,t) = \phi(x,y) \chi(x,t)$$
$$p'(x,y,t) = f(x,y) \chi(x,t)$$

where $\chi(x,t) = e^{i\theta(x) - i\omega t}$ in which $d\theta(x)/dx = \alpha(x)$

According to ref. 18 this decomposition is appropriate when the flow has the following properties:

- The velocity profiles, wavelengths and growth rates change slowly in the stream wise direction (second and higher derivatives in stream wise direction may be neglected
- The disturbances grow and decay as convected instabilities ( for explanation of this there is only a literature reference in ref. 18)

Using these assumptions and the decomposition above one obtains the so-called Parabolized Stability Equations (PSE). The Parabolized Stability Equations constitute an initial – boundary value problem which means that in addition to the conventional boundary conditions (listed several times on present study) an initial condition is also needed. The initial condition is obtained by solving the local stability problem (parallel primary flow case).
When implementing the PSE-equations to the two layer case of air – liquid, Boelens and Sijp adopts the liquid film height of Nelson et al with a constant liquid volume flux of $Q \ (m^2/s)$. However the film height is decomposed to primary flow and disturbances as:

$$h(x,t) = H(x) + h'(x,t) \text{ where } H(x) = H_0 \left(\frac{x}{x_0}\right)^{1/4}$$

The velocity profile in the gas boundary layer is calculated using the Blasius equation:

$$f'''(\eta) + 0.5 f(\eta) f''(\eta) = 0$$

with similarity coordinate of $\eta = H(x)\left(\frac{U_x}{(\nu_g x)^{1/2}}\right)$. $U_x$ is air free stream velocity.

Numerical results have been calculated with a liquid having properties close to anti-icing fluid (dynamic viscosity ratio to air = 50, density ratio = 1000). The liquid layer initial height is 1 mm and air free stream velocity is 40 m/s.

The results of a numerical solution of PSE are shown in form of real and imaginary parts ($a_r, a_i$) of non-dimensional complex wave number $a(x) = i\alpha(x)$ vs $x \ (m)$ in figures 29 - 33. The first two figures (29-30) describe the Tollmien Schlichting mode and the latter three figures describe the interfacial modes. As a conclusion the Boelens and Sijp state that both the Tollmien – Schlichting mode and the interfacial mode that appear in temporal stability analysis (parallel flow) also appear in their spatial stability analysis. For spatial stability analysis however the interfacial mode is dominant mode for de/anti-ice fluids, in contrast to what has been found in temporal analysis where the T-S mode is the dominant mode. The word “dominant” is left unexplained in ref. 18.

![Figure 29. Tollmien Schlichting mode: $a_r$ vs $x \ [m]$. Full line: temporal analysis. Dotted line with dots: spatial analysis](image-url)
Figure 30. Tollmien Schlichting mode: $a_i$ vs $x$ [m]. Full line: temporal analysis. Dotted line with dots: spatial analysis

Figure 31. Interfacial mode: $a_i$ vs $x$ [m]. Full line: temporal analysis. Dotted line with dots: spatial analysis

Figure 31. Interfacial mode: $a_i$ vs $x$ [m]. Full line: temporal analysis. Dotted line with dots: spatial analysis
Another conventional application of linear stability theory with temporal stability formulation for the problem of flat plate with liquid fluid on it is presented by Özgen et al. in ref. 19 (1998). The fluid however is a general one in the meaning that they include also non-Newtonian cases with the power law viscosity in their solution. The basic Orr-Sommerfeld equation is derived for air and liquid in the traditional way with the boundary conditions listed many times previously in the present literature study. The continuity of normal stress on the free surface includes the effect of gravity and surface tension. This is depicted in the equations by the Froude and surface tension parameter (S). The velocity distribution assumptions are – as in many previous studies:

- Air boundary layer: Blasius boundary layer
- Liquid boundary layer: linear velocity distribution

As the liquid layer viscosity permits the non-Newtonian liquids the final Orr-Sommerfeld equation for the liquid includes also the exponent n from power law. Value n=1 means Newtonian fluid.

The merit of ref. 19 is its numerous calculation cases presented for a variety of different parameters. To understand the results there is a list of used parameters below – suffix 1 refers to air and 2 to liquid.

\[ m = \frac{\mu_2}{\mu_1} \]
\[ r = \frac{\rho_1}{\rho_2} \]
\[ R = \text{Reynolds number} = \frac{Ud_1}{(\mu_1/\rho_1)} \text{, where } d_1 = \text{Blasius length scale that is not explained anywhere on the text – probably } (ux/U)^{1/2} \]
\[ F = \text{Froude number} = \frac{U}{(gd_1)^{1/2}}. \text{ A used parameter in the results is } F^2 = gd_1/U^2 \]
\[ l = d_2/d_1 \text{ where } d_2 \text{ is the film thickness of liquid and } d_1 \text{ the above mentioned Blasius length scale} \]
\[ a = \text{dimension less wave number} = \frac{2\pi}{\lambda} \text{ where } \lambda \text{ is made dimensionless with Blasius length scale} \]
\[ c = \text{complex wave speed} = c_r + ic_i \text{ in which the real part denotes the propagation speed (phase speed) and the imaginary part denotes the amplification factor.} \]
flow is unstable when $c_i > 0$, stable when $c_i < 0$ and neutrally stable when $c_i = 0$. The parameter $\sigma_i = \alpha c_i$ is called the amplification rate.

The instability modes are divided to Tollmien Schlichting (TS) mode (hard mode) and interfacial (Yih) mode (soft mode). According to Özgen et al. the interfacial mode is due to viscosity stratification alone. Thus the first case considered is equal dense fluids ($r = 1$) with $m = 10$, $l = 0.5$ and $S = 0$. This case is described in fig 33 and 34.

Figure 33. Neutral stability curves TS-mode (thick line) and interfacial mode (thin line). $m = 10$, $r = 1$, $l = 0.5$, $S = 0$

Figure 34. Amplification factor of TS mode (thick line) and interfacial mode (thin line). $m = 10$, $r = 1$, $l = 0.5$, $S = 0$ and $R = 4000$

It can be seen from fig. 33 that there are regions at $R$-$\alpha$ plane where TS mode is stable but interfacial mode is unstable. The difference of these modes are more clear ($R = 4000$) in fig. 34 which implies that the TS mode is likely to be observed first except at low Reynolds numbers.

The effect of increasing viscosity on the liquid layer is seen in figure 35 for TS mode. As the viscosity of the liquid increases ($m$ increases) the neutral stability curve shrinks and in the limit approaches the curve for a single layer flow without the liquid.
Figure 35. Effect of increasing the viscosity stratification on the neutral curves of TS-mode (m= 10, 100 and 1000 (increasing from out to in)). The solid line (innermost) is the single layer flow without liquid. (r = 1, l = 0.5 and S=0)

In fig. 36 are the amplification factors for both TS and interfacial modes with growing viscosity stratification. At smaller values of m the amplification factors are larger and enclose larger area of $c_1 – \alpha$ plane.

Figure 36. Effect of increasing viscosity stratification (increasing m) on the amplification factors of the TS mode (thick lines) and interface mode (thin lines) at R = 4000. Lines: -- m = 10, -- m= 100, ... m = 1000. (r =1, l =0.5, S =0)

In other words the magnitudes of the amplification factors of the TS mode approach those of the single layer (no liquid) flow and they vanish for the interfacial mode as m increases. Therefore in the limit m $\rightarrow \infty$ we have the eigenvalues belonging to the TS mode which are identical to those for a single layer flow, and eigenvalues belonging to an interfacial mode that are asymptotically neutrally stable. The conclusion is that, as m $\rightarrow \infty$ the lower layer remains idle in terms of contribution to the stability characteristics of the flow. Since m for anti-icing fluid air combination is very large ($\approx 5 \times 10^5$) it is the TS mode that manifests itself first.
Özgen at al comments on Yih’s paper considering the comparison between his theory and experiments of ref. 3.: “Yih’s quick computation for the extrapolated conditions of ref. 3 contains a numerical error leading to a spurious result that the mode observed here is a soft mode (=interfacial mode). In fact putting in the right data and performing the arithmetic gives that the mode observed is TS mode.

To continue the effect of viscosity on the interfacial mode there is variation of m from 2 to 10 described in fig. 37. The other parameters are as in figure 36. The effect of variations in m to both modes (TS and interfacial) at very low wave numbers (long waves) is negligible. However we see a considerable growth in the stable region of the soft (interfacial) mode at moderate to high wave numbers (short waves) as the viscosity ratio m decreases. According to some other researchers (referred by Özgen et al) the stable region occupy the whole domain when m→1, leaving the Tolmien Schlichting mode the only unstable mode.

The effect of increasing the liquid film thickness is seen in figure 38. Making the liquid layer thicker has almost no effect on the region of very small wave numbers (long waves). At moderate and high wave numbers (short waves) regions where both modes are stable (TS and interface mode) expands when liquid layer thickness increases. In the case for l = 2.0 the unstable region for the TS mode has almost disappeared and now a large stable region is present for R>150 in the upper domain.

Figure 39 demonstrates the effect of surface tension for the interfacial mode. The effect of the surface tension is more important for the short waves (large wave number). The neutral stability curves for long waves remain unaltered – the curve is identical for all three surface tension cases presented. When S ≠ 0 a cutoff wave-number value is apparent above which the flow is unconditionally stable for the interfacial mode. For short waves the curvature of the free surface is higher than for long waves. This explains why the surface tension which is directly proportional force to the curvature of surface has the stabilizing effect in the interface mode.

Figure 37. Effect of mild viscosity stratification on neutral stability curves of TS mode (thick lines) and interfacial (thin lines) mode. Lines: solid line: m = 10, …..: m=5, ---: m=2. (r=1, l=0.5, S=0)
Figure 38: Effect of thickness of the liquid layer on the stability curves of TS (thick lines) and interfacial (thin lines) modes. Lines: solid line: $l=0.5$, ....: $l=1.0$, ----: $l=2.0$ ($m=10, r=1, S=0$)

Figure 39. Effect of surface tension on the neutral stability curves of the interfacial mode. Lines: solid line: $S = 0$, .....: $S = 1.0$, -----: $S = 5.0$. ($m=10, r=1, l=0.5$).

Figure 40 supports the previous argument. It shows that the amplification factor $c_i$ asymptotes to a finite negative value which suggests that beyond a certain value of the wave number the damping effect of the surface tension does not increase with increasing interface curvature.
Figure 40. Effect of surface tension on the amplification factors of the interfacial mode. Lines: solid: $S=0$, ....: $S=1.0$, ----: $S = 5.0$, $(m=10, r =1, l =0.5, R = 4000)$

Figure 41 presents the effect of gravity in form of inverted square of Froude number ( $F^{-2}$ ) for the interfacial mode at $R = 4000$. Both the surface tension and the gravity terms appear in Orr Sommerfelds equation system only in the normal stress matching condition at the interface with same signs, so their effects should be similar which is supported by the figure.

Figure 41. Effect of gravity on the amplification factors of the interfacial mode. Lines: solid: $F^{-2} = 0.001$, ....: $F^{-2} = 0.01$, -.--.-: $F^{-2} = 0.1$. $(m =10, r =10, l =0.5, S =0, R = 4000)$

The effect of density ratio at constant Froude number is presented in fig. 42. Density stratification does not have a monotonic effect. It destabilizes the flow
until a value of r around 5 and then starts stabilizing it. The stabilizing effect becomes more pronounced as the density ratio increases. For values of r greater than 10 there is a stable region above wave number of around 5.

Figure 42. Effect of density ratio on the amplification factors of the interfacial mode. Lines: solid: r = 2, -----: r = 5, ————: r = 10, __ __ : r = 20, -.-.-.-.-.: r = 50. (m = 10, l = 0.5, S = 0, F \(2\) =0.001, R = 4000)

Figure 43 demonstrates the effect of non-Newtonian behavior of the lower fluid. Decreasing n means increasing deviation from Newtonian behavior while value n=1 means a Newtonian fluid. The shape of the neutral stability curve for the interfacial mode at small wave numbers remain the same. However the stable pocket of the interfacial mode at high wave numbers disappears immediately when n≠1 according to Özgen et al. According to ref. 19 this curve is very sensitive to other parameters of the problem as the authors of ref. 19 were able to capture it only for some combinations of the parameters. They claim the waves in this wave number range would be damped out by surface tension and that the curve is of mathematical nature only. The question of course arises that as S = 0 in all the cases how can surface tension damp out anything? The shape of TS mode curves seems not to change either. The first impression is that non-Newtonian power law nature of the liquid does not effect on stability characteristics.

However the amplification factor figure (fig. 44) gives quite the opposite impression. Amplification factors for the TS mode reduce significantly with decreasing n. At a value of n = 0.2 the maximum amplification factor has reduced to half of the value at Newtonian case n=1. A similar effect is evident for the interfacial mode as well. The amplification factors decrease progressively with decreasing values of n and at a value of n = 0.2 they vanish. The behavior is similar to the behavior with increasing viscosity ratios m. The explanation for this is somewhat complicated as value m does not have a simple meaning of viscosity ratio once n≠1 in the paper of Özgen et al. The definition of m is m = \(\frac{k_2}{\mu_1}(\frac{U_\infty}{d_1})^{n-1}\) where k_2 is consistency factor defined with equation \(\mu = kI^{(n-1)/2}\) where I is an invariant of the rate of strain tensor defined as I = 2 \(e_{lm}e_{lm}\) where \(e_{lm}\) is rate of strain tensor. In case of n = 1 only the parameter m reduces to simple viscosity ratio: m = \(\frac{\mu_2}{\mu_1}\). However there is the following explanation given in ref. 19 for the resemblance of increasing m and decreasing n: “This means that although the viscosity ratio m remains the same, the boundary layer shears an effectively more viscous fluid at smaller values of n. This is why the effect of decreasing n has the same effect as increasing viscosity ratio m.
Figure 43. Effect of non-Newtonian liquid layer on the neutral stability curves of the TS mode (thick lines) and interfacial mode (thin lines). Lines: solid: $n = 1$ (Newtonian), ….: $n = 0.8$, - - - : $n = 0.5$, -.-.-.-: $n = 0.2$. ($m=10$, $r = 1$, $l = 0.5$, $S = 0$)

Figure 44. Effect of non-Newtonian liquid layer on the amplification factors of TS mode (thick line) and interfacial mode (thin line). Lines: solid: $n = 1.0$ (Newtonian), ….: $n = 0.8$, - - - : $n = 0.5$, -.-.-.-: $n = 0.2$. ($m=10$, $r =1$, $l = 0.5$, $S = 0$, $R = 4000$).

The paper of Özgen et al includes still couple of more cases of stability with different parameters such as viscosity ratios of less than one and density ratio less than one which does not have other than academic interest in the present
Though ref. 19 includes a lot of different cases there is one clear shortage in them. The values are far from the case of real interest: air and anti-icing fluid. In this case the parameters $m$ and $r$ should be around the following: $m = 10^5$, $r = 10^3$.

Though the gravity consideration in form of Froude numbers is interesting it is hard to see the gravity to change in practice. Of course the air velocity outside the air boundary layer affects also to this value (!).

The next chapter in ref. 19 is a treatment of a classical verification experiment of ref. 3 that was used initially by Yih in ref. 1. Using the data copied out from Yih’s paper Özgen et al has calculated in ref. 19 the amplification rate of both TS mode and interfacial mode. They are presented in fig. 45. They also claim to have corrected Yih’s calculation for wave number. According to ref. 19 Yih’s wave number is non-dimensionalized with a wrong reference length. After this correction they note that the mode Yih found is TS mode not interfacial as Yih claims.

Another support for the argument of Özgen et al that the waves seen are TS not interfacial mode waves is seen in fig. 45 as the amplification rate maximum of TS mode is 3 orders of magnitude larger than for interfacial mode.

The results of calculations of ref. 19, ref. 1 (Yih) and some other studies has been summarized in table 4. This table includes peculiarities considering Yih’s results. The difference in wave number had its explanation in “wrong” reference length however the amplification factor $\sigma_i$ and a new parameter $c_r - U_o$ which is the non-dimensionalized difference between wave (phase) speed and the interface speed (liquid speed on free surface) does not match with results given by Yih. A straightforward calculation from Yih’s paper gives $\sigma_i = 5.02 \times 10^{-6}$ instead of $4.58 \times 10^{-6}$ given in table and for $c_r - U_o = 1.26 \times 10^{-5}$ instead of $2.08 \times 10^{-5}$.

Figure 45. The amplification rates for TS mode (thick line) and interfacial mode for the case treated in ref. 1 (based on measurement of ref. 3). Note that the arrows on curves point to the applicable scales (3 orders of magnitude difference).
Table 4. Comparison of the results of the case treated originally in ref. 1.

<table>
<thead>
<tr>
<th>Author</th>
<th>$\sigma_{\text{low}}$</th>
<th>$\alpha_{\text{up}}$</th>
<th>$\sigma_{\text{L,max}}$</th>
<th>$\alpha_{\text{L,max}}$</th>
<th>$(\sigma_{\text{L}} - U_{\text{L}})\sigma_{\text{L,max}}$</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yih</td>
<td>0.0040</td>
<td>0.3666</td>
<td>0.2008</td>
<td>4.58 x 10^{-6}</td>
<td>2.08 x 10^{-3}</td>
<td>interfacial</td>
</tr>
<tr>
<td>Boelens et al.</td>
<td>0.0054</td>
<td>0.3767</td>
<td>0.2057</td>
<td>5.17 x 10^{-6}</td>
<td>2.64 x 10^{-3}</td>
<td>interfacial</td>
</tr>
<tr>
<td>Present study</td>
<td>0.0053</td>
<td>0.3754</td>
<td>0.2022</td>
<td>5.13 x 10^{-6}</td>
<td>2.55 x 10^{-3}</td>
<td>interfacial</td>
</tr>
<tr>
<td>Hendickson et al.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.1402</td>
<td>n.a.</td>
<td>n.a</td>
<td>TS</td>
</tr>
<tr>
<td>Present study</td>
<td>0.0913</td>
<td>0.2008</td>
<td>0.1504</td>
<td>2.06 x 10^{-3}</td>
<td>0.3449</td>
<td>TS</td>
</tr>
</tbody>
</table>

The last chapter of ref. 19 deals with turbulent velocity profile for gas and its effect on instability modes. Özgen et al. refers to Miesen and Borsma work (ref. 20) which is an extensive study on the subject including several different theoretical and experimental cases as verification material. In ref. 20 the flow configuration studied is a vertical channel with liquid film on the wall and a concurrent airflow (blowing upstream). Though this is not the exactly the flow configuration of interest in present review the results considering the instability of the liquid layer are valid for a horizontal flat plate as well.

Miesen and Borsma finds that in case of turbulent gas boundary layer there is two instability modes, the interfacial mode and the internal mode. They show that the viscous sublayer of the turbulent boundary layer of the air is critical for the interface mode whereas it is in the liquid layer for the internal mode. Özgen et al compares in several tables results of their own and results listed in Miesen and Borsma that are beyond the scope of this review however there is one particular case of interest. It is once again the experiment of ref. 3 that was originally used by Yih, but this time applied with a turbulent air boundary layer. All the other parameters of the flow are as presented earlier however now Özgen et al select the shear stress at the interface to be $\tau = 10.0 \text{ Pa}$. There is no reasoning behind this value in ref. 19. The motivation in this particular study is according to Özgen et al to clarify whether the waves in the photographs of ref. 3. (see figure 1) could have been created by a turbulent profile or not. After their new calculations the interfacial mode yielded a maximum amplification rate of $\sigma_i = 9.2 \times 10^{-4}$ at $\alpha = 2.93$ based on liquid height (=1.1 mm which implies a wave length of 2.4 mm). The propagation velocity $c_r - U_o$ was found to be $8.66 \times 10^{-5}$. The wave number calculated from the measurements of ref. 3 was found to be approximately $\alpha = 0.5$ (meaning a wave length of approximately 12 mm). According to Özgen et al the large difference between these findings suggests that the waves observed in the experiments (of ref. 3) are formed by a laminar boundary layer and possibly of TS-type.

There appears now a clear contradiction once again between the selected shear stress and the interpretation of the results. There is no clear evidence why the boundary layer would not be laminar. The local $Re$ – number at the point where the parameters have been calculated is 460 000. In case of clean surface the boundary layer may well be laminar. With roughness of figure 1 this is not more so evident. However what is clearly contra dictionary is the selected shear stress value of 10 Pa. If the boundary layer is claimed to be laminar the friction coefficient should be approximately the Blasius value of $C_f = 0.664 / (Re)^{1/2} = 0.0011$. There is a order of magnitude difference to the given value.

Boelens and Hoejmakers (ref. 21) studies the formation of waves on the interface of a two fluid stratified channel flow configuration solving directly the Navier-Stokes equations in both fluids. This 2D time dependent CFD-method uses artificial compressibility (local time stepping) and multi gridding to solve N-S-equations on a grid moving in y-direction (see figure 46).The motion of the grid is determined by the kinematic interface condition. A special procedure has been
developed to incorporate the conditions to be satisfied at the interface directly into the finite volume scheme. The CFD procedure is based on Jameson’s method introduced in ref. 22.

The CFD – method of Boelens and Hoejmakers is tested in ref. 21 for the transient single fluid Couette flow between two flat parallel solid walls, the Stokes oscillating flow between two flat parallel solid walls and the transient two-layer Couette coutte flow between two flat, parallel solid walls. Note that though the two first are considered as stratified flows the fluid is is single in the meaning that across the interface (fig. 46) there is no density or viscosity difference and the surface tension is $= 0$. The third case the only genuinely two layer problem where there is viscosity difference between the layers though the densities are equal. All the test cases of ref. 21 are far from the interest of this review however Boelens and Hoejmakers have in their report shown that this kind of CFD analysis for stratified flow with an interface is possible. As a matter of fact Boelens and Hoeijmakers comment their work as follows: “Presently (1999) calculations for a channel flow configuration of air shearing a water layer are being performed. The resulting wave shapes and growth rates will be compared to those available in the literature. In the near future also calculations for the channel flow configurations of air shearing a de/anti-icing fluid will be performed. Finally calculations have to be (!) performed for the configuration of the stratified flow of two fluids of different viscosity and density, one bounded by a solid wall and one extending to infinity.” An example of results in ref. 21 is shown in figure 47 (Couette –Couette flow).

In ref. 23 Rothmayer and Tsao presents a further developed theory of the triple-deck method presented by Tsao et al in ref. 12. The flow configuration under consideration is a water film running on an airfoil surface but it is applicable for at least Newtonian de-ice fluids too. They end up with number of reasonable principles considering the mechanism of mass flux of the water film, rather than futile speculations on instability modes which seems to be a standard in this branch.

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![Diagram](image-url)  
**Figure 46.** The computational domain of CFD method presented in ref. 21.
Figure 47. Nonlinear wave formation for Couette Couette flow. Viscosity of lower layer is 5 times the value of upper layer. Densities in both layers equal, $We = 0.01$ (surface tension taken into account)

The basic equations are once again the familiar conservation equations of N-S equation system with familiar boundary conditions and surface interface kinematic conditions. The flow configuration and the local coordinate system applied (with $x$ aligned with prevailing surface shear direction) are presented in figures 48 and 49.

Figure 48. Schematic diagram of the surface water contamination problem.
Figure 49. Schematic diagram of the boundary layer coordinate system.

A number of non-dimensional parameters are found to control the above equations. These include:

- **Re – number:** \( \text{Re} = \frac{\rho_{\infty} V_{\infty} L}{\mu_{\infty}} = 10^3 \ldots 10^7 \gg 1 \)

- **Water to air viscosity ratio:** \( M = \frac{\mu_{\text{water}}}{\mu_{\infty}} = 100 \ldots 200 \gg 1 \)

- **Air to water density ratio:** \( D_{aw} = \frac{\rho_{\infty}}{\rho_{\text{water}}} = 10^{-4} \ldots 10^{-3} \ll 1 \)

- **Dimensionless parameter without name (see fig. 51):** \( M = M D_{aw}^{1/2} \)

- **Non dimensional surface tension number:** \( \sigma = \frac{\sigma^*}{V_{\infty} \mu_{\infty}} = 22\ldots 89 \), where \( \sigma^* \) = dimensional surface tension

The water film is assumed to lie on a smooth surface on which an air boundary layer of thickness \( \text{Re}^{-1/2} \) has formed. The disturbance free case that will be termed the “flat film” in which a film of known depth lies on a locally flat surface (flat means no waves or other distortions). This flat film generates a perturbation that appears as a higher order effect – too small to impact the primary scales of the study.

The boundary layer theory of the ref. 23 is based on the surface roughness theory of eg Smith et al. (ref. 24). This boundary layer theory assumes that a roughness element of known length scale \( \Delta \) is positioned at some point within an attached Prandtl boundary layer of thickness \( \text{Re}^{-1/2} \). In the theory of Smith et al the stream wise length scale is assumed to be some function of Re-number as Re-number becomes large. A similar approach was used by Tsao et al. (ref. 12) to describe the instabilities on thin films. In this study however the stream wise length scale was fixed at the value of \( \text{Re}^{-3/8} \), which gives the triple deck (see Stewartson – ref.13). The study of ref 23 relaxes this assumption and allows the stream wise length scale to float through the allowable values of ref. 24 structures. The exact value of the stream wise length scale is determined from the assumption that the air shear and pressure are in balance with water – air surface tension on the air sublayer scales of ref. 24. The most important issue that comes out of this new
theory is its ability to describe the cross-over from shear driven to pressure driven films. The theory describes four different stages (I, II (cross-over stage), IIIa and IIIb) of liquid film flow as a function of film thickness. Figures 50 and 51 describes schematically this division to stages. Stage II is the cross-over (from shear driven to pressure driven films) stage.

After a lengthy scale analysis based derivation Rothmayer and Tsao end up to the following characteristic stream wise ($\Delta$) and vertical ($y$) length scales for the stage II:

$$\Delta \sim \sigma^{3/7} \text{Re}^{-9/14}, \quad y \sim \text{Re}^{-5/7} \sigma^{1/7}$$

(4)

These scales are essential to interpret the results of ref. 23 in dimensional variables. What actually is found by these scales according to Rothmayer and Tsao is the characteristic film thickness that bounds shallow shear driven waves and the deep pressure driven waves. Shallow and deep are relative terms as they both are a fraction of the air boundary layer thickness.

The linear stability analysis in ref. 23 is done in 3 dimensions. The wave numbers are in x- and z- direction:

$$\alpha = 2\pi / \lambda_x \quad \beta = 2\pi / \lambda_z$$

Wave lengths above are dimensionless so that (eq. (4)) dimensional $\lambda_x^* = \sigma^{3/7} \text{Re}^{-9/14} \lambda_x L$. Respectively dimensional values for film thickness is $f = \text{Re}^{-5/7} \sigma^{1/7} h L$, where $h$ is the dimensionless film thicknesses. Note that this applies to Stages II only.

Some results of instability analysis for 2 dimensional cases ($\beta = 0$) in Stages II and III are shown in figures 52, 53 and 54.

The surface tension stabilizes the waves at short wavelengths due to high curvature values at short wave lengths as shown in earlier references too on this review.

Figure 50. Schematic diagram of the Stage II air sublayer and water film for the critical cross-over film height. The stream wise wavelengths of the interfacial waves are close to the boundary layer thickness and the critical film height is a small fraction of it.
Figure 51. Schematic diagram of the Stages in ref. 23. In the Stage II, the wave amplitude is the same as the film depth. In all other stages, the wave amplitude is smaller than the film depth.

Atypical solutions with varying scaled film heights $h$ are shown in figure 53. The figure shows that the growth rate is increasing with increasing film thickness, while the wavelength at maximum growth rate is approaching a constant. At small film thicknesses the growth rate is decreasing and the wavelength is becoming short. The critical film thickness is the cross over between these two behaviors, and that is claimed in ref. 23 to occur for $h$ approximately in the range 0.1 to 100 (though the maximum $h$ in the figure is 50?).

The maximum growth rate or neutral wavelength has the following proportionalities (up to the upper limit of $h$-values in fig. 53.):\[
\lambda_x = \frac{h^{3/8}}{(\eta_{\text{air}})^{1/4} \lambda^{1/2}} \quad (5)
\]
where $\lambda$ is a dimensionless skin friction parameter. With increasing film thickness the instability wave length approaches a constant.
Figure 52. Instability growth rate for a typical linearly unstable solution in Stage II with and without surface tension.

Figure 53. Typical effect of film thickness on the instability growth rate in Stage II. The maximum growth rate curve and the limit solution for large film thickness is also shown. Dimensionless film thicknesses are $h = 0.05, 0.13, 0.97, 2.6, 6.9, 18.6, 50.0$. 
A detailed examination of the eigen relation of the two limiting film thickness regions reveals that all the pressure forcing drops out of the dispersion relation (between wavelength and film thickness) at the leading order in the limit of small film thickness. At large film thickness all the shear forcing drops from the leading order dispersion relation. The proportionality (5) indicates also that in the small waves region the instability wavelength decrease with increasing shear stress in the air.

The above presented may be interpreted as follows (quote from ref. 23): “Given ambient flow conditions and film properties (viscosity, surface tension, etc) the y-scaling of equations (4) gives the film height at which a cross-over from shear driven to pressure driven waves occurs. The x-scaling (Δ in eq (4)) is the stream wise wavelength of those waves. For film thickness less than this critical value, the mass flow within the film will be driven by the air shear stress. For film thickness greater than the critical value the mass flow within the film will be driven by air pressure gradient.”

Rothmayer and Tsao gives in ref. 23 the following welcomed comment on liquid massflux: “Of central concern for practical applications involving aircraft icing and de-icing is the rate at which water, or another liquid is transported along the aircraft surface.”
ref. 23 divides the liquid mass flux in two parts depending on the Stage considered (I or II): the mass flux for a flat-film Couette flow \( \dot{m}_{\text{flat}} \) (Stage I) and the time dependent mass flux of wavy liquid film \( \dot{m}_x \). Computing the total mass that passes through the plane in one temporal period of the wave and comparing this to the mass of a flat film in the same interval, Rothmayer and Tsao ends up to scaled mass flux parameter of:

\[
F = \frac{\langle \dot{m}_x \rangle}{\langle \dot{m}_{\text{flat}} \rangle},
\]

where the averages are taken in time at a fixed \( X_0 \) position over one temporal period of the wave. This ratio of mass flux variation (\( F \)) with liquid film thickness (and different Stages) is shown in Figure 54.

To summarize some of the results above a quote from the abstract of ref. 23: “When the film is this, surface waves are driven by the air shear stress. These shear driven waves do not significantly effect (affect) the mass flux within the film (Stage I). When the film exceeds a certain critical thickness which is small percentage of the boundary layer thickness, then air pressure gradients drive the wave motion (Stage II, IIIa, and IIIb). Pressure driven waves are found to control mass flux within the film.”

In their analysis of wave driving mechanisms, Rothmayer and Tsao are the first ones to give an answer to “too” high values of shear stress and friction coefficient values of other authors mentioned in this review. The abnormal high shear values should probably be replaced with pressure forces. When considering eg. the Schlichting formula for friction coefficient (equation (3) or fig. 3.) there is a limit in roughness element size for its use – it is obvious that after some size there will be as well pressure forces acting in the direction of shear stress.

As the validation case of Boeing measurements (ref. 3.) used first by Yih (ref. 1) seems to have grown to a necessity in the studies of this branch, Rothmayer and Tsao take naturally part to the competition. The dimensionless parameters in their theory have been calculated for this measurement case as follows:

- \( M = 598802 \)
- \( D_{aw}^{-1} = 972 \)
- \( \# = 19206 \)
- \( Re_x = 460248 \)
- \( \sigma = 68.7 \)
- \( \lambda = 0.3833 \) (Blasius B-L)

With these parameters they get as a Stage II scaled film thickness of \( h = 29.35 \). This film thickness is according to Rothmayer and Tsao within the Stage II range “but leaning towards Stage III”. They calculate a value for maximum growth rate \( \lambda_x = 20.9 \). Note that fig. 53 is not directly applicable now as it is based on different value of dimensionless skin friction parameter \( \lambda \). A dimensional wavelength is therefore \( \lambda_x = 20.9 \Delta L = 6.8 \text{ mm} \).

The dimensionless wavenumber calculated in ref. 23 for this case is \( \alpha = 1.02 \) which should be compared with Yih’s value of 0.33 or the experimental value of 0.5. The discrepancy is quite large and thus Rothmayer and Tsao gives explanations for it (quote of ref. 23): “the fact that we are using a first order asymptotic theory that must be corrected to include higher order effects; the use by us and Yih of Blasius values for airfoil boundary layer profiles which may be turbulent; the difficulty in measuring the wave free film thickness that would exist at \( \frac{3}{4} \text{ chord} \); the use of a piecewise linear boundary layer by Yih.”
As the difference in wavelengths is 200% it the first explanation is unlikely. So is the third one as there is no difficulties to estimate the thickness within say 20%. The second explanation would likely deviate as well Yih’s as Rothmayer and Tsao’s results in same direction of same order of magnitude.

There is a good summary in ref. 23 of Rothmayer and Tsao’s theory’s main results which are quoted directly in the following:

1. There is a critical film thickness below which the film surface waves are driven by the air shear stress. Above the critical film thickness the surface waves are driven by air pressure.

2. The cross over from shear driven to pressure driven waves always occurs when the film thickness is a small fraction of the boundary layer thickness (roughly 5-40%).

3. On the cross-over scale of Stage II the film is inertial (i.e. unsteady and convective effects are important within the film). Using lubrication approximation within the film will likely result in about 10% error. The lubrication approximation is valid for computing mass flow for film thicknesses which are much less than the critical value.

4. For film thicknesses much less than the critical value the surface water mass flow may be computed from the Couette flow within the film that is generated by the mean air shear stress.

5. For film thicknesses larger than the critical value, the mass flux within the film is determined at leading order by the wave motion on the film surface and is larger than the mass flux given by a shear driven Couette flow at the same film thickness.

6. If a lubrication approximation is used on the cross-over scale, then the approximation will fail for water film thicknesses that are slightly larger than critical, due to the fact that unsteady effects will quickly come into play within the film as the thickness increases.

7. For all cases computed in this study, interfacial waves increase mass flux within the film.

In their second report on the issue Özgen et al. (ref. 25, 2002) study the de/anti-icing fluid behavior in airflow both theoretically and experimentally. This time the theoretical approach includes a turbulent air boundary layer. The results of theory and experiments are then compared.

The experiments of ref. 25 was done at von Karman Institute Cold Wind Tunnel – 1 facility (CWT-1). VKI CWT-1 is a wind tunnel with a quite small test section of 0.1m x 0.3m. The length of the test section is 1.6 m (fig. 55). The equalities in dimensions with the de/anti-icing test wind tunnel at AMIL, Canada, are not inadvertent. The Aerodynamic Acceptance Test procedure was partly developed at VKI CWT-1 which led to the dimensions of AMIL AAT facility. The maximum speed in test section is 70 m/s.

The dimensions and structure of the wind tunnel is not mentioned in ref. 25. This may give reader a false impression that the fluid behavior is experimented on a flat plate although it is strictly speaking done in a channel flow. There is an analysis of boundary layer merging in a tunnel of this size in ref. 26.
The experiments in ref. 25 are focused in wave formation and behavior of de/anti-icing fluids in airflow. The measurement technique used is so called Light Absorption Technique (LAT) which correlates the thickness to the transmitted light intensity. When light is incident on a medium, only a part of its intensity is transmitted to the other side. There is a loss of intensity due to reflection (spurious effect) and absorption (useful effect). An observer positioned above the medium sees an intensity inversely proportional to the length of the path of the light inside the medium. The relation between the incident light and transmitted light is:

$$I = I_0 \exp(-\varepsilon^*d^*)$$

where $I_0$ is the incident light intensity, $I$ transmitted light intensity, $\varepsilon^*$ the light absorption coefficient and $d^*$ the medium thickness. Knowing $\varepsilon^*$ from a suitable calibration the medium thickness may be determined by measuring the transmitted light intensity.

The experimental setup of ref. 25 is presented in fig. 56.

Figure 55. VKI CWT-1 wind tunnel

Figure 56. Experimental setup of ref. 25.
The fluid in the test section is dyed with a violet dye to increase the image contrast. For the same reason a green filter is attached to the lens of the CCD camera.

For calibration purposes a wedge shaped glass container is utilized. This allows the correlation of the measured light intensities with corresponding fluid thickness (note the linear change of fluid thickness compared to stepwise change in grooved calibration containers in several other studies). Two calibration images is needed before test: first with the empty container in the test section and the fluid filled one. To get the effect of the fluid only the first one is subtracted from the second one. A MATLAB code calculates the absorption coefficient from the RGB-values of the CCD images. Figure 57 presents the two mentioned calibration images. Özgen et al. claims to achieve an accuracy in film depth of 0.1 mm using this technique.

The waves during the test are analyzed from separate video frames. After processing the RGB-values of an image to intensities a thickness variation with longitudinal position in the test section is produced. Figure 58 gives an example of how the single video frame is transformed to a graphical representation of waves. The wavelength is calculated from a graph as in figure 58 (b) after performing a
fast Fourier transform to the data. For the calculation of the wave speed a rectangular portion is chosen on a frame and the code chooses automatically the same portion from sequential frames. After calculating cross correlation between sequential images the wave speed may be determined. For fairly two dimensional waves the same thing can be done with two individual cuts with less time consumed.

The statement of accuracy in wave speeds in ref 25 is odd as accuracy is said to be 6 mm/s for 12 frames/s and 0.1 mm/s for 0.25 frames/s. Should it not be the other way around?

The theoretical approach of ref. 25 is basically identical with ref. 19. The only difference is that now the air boundary layer considered is turbulent instead of the Blasius boundary layer in ref. 19. The velocity gradient in the liquid is linear as usual. The turbulent air boundary layer is divided into two parts:

\[ u(y) = (u_r^2 / u) \quad \text{for} \quad 0 \leq y \leq s \left( u / u_r \right) \text{ where } s = 5-8 \]  
\[ u(y) = ay^c + b / y^2 + d \quad \text{for } y > s \left( u / u_r \right) \]  

The first part is the so called laminar sublayer where \( u_r = (\tau_o / \rho)^{1/2} = \text{friction velocity} \) and \( \tau_o = (1/2) \rho U^2 C_f = \text{shear stress at the interface} \).

The parameters \( a, b \) and \( d \) in the outer part expression (6b) of the air boundary layer are determined by the continuity of velocity, its first and second derivatives at laminar sublayer outer edge \( y = s \left( u / u_r \right) \). Parameter \( c \) is found by satisfying the free stream boundary condition. The motivation for the outer part velocity expression is explained referring to other studies alike. Notice that the mean velocities in equations (6) does not take the turbulent fluctuations into account.

The linear instability equations (Orr Sommerfeld) are equal to the ones in ref. 19. One of the objectives of the experimental work in ref. 25 is said to be validating the linear stability code developed at VKI. As the code requires the rheological data of the fluids the densities, viscosities and surface tension coefficients of fluid tested were measured. Two Newtonian (deicing, M and D) and one non-Newtonian (anti-icing, A) fluids were used for tests. The properties of fluids tested are collected to table 5. Viscosities were measured with a Brookfield DV-II viscometer. As for the non-Newtonian fluids the viscosity varies with shear rate as shown in figure 59 there is the closest curve fitting according to the power law expression (see equation (2)) given as a viscosity (column 5) in table 5. The surface tensions in ref. 25 were measured with Kruss K8 interface tensiometer.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Type</th>
<th>Dilution rate</th>
<th>( \rho^* (kg/m^3) )</th>
<th>( \mu^* (cps) )</th>
<th>( \gamma^* (mN/m) )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1030</td>
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<td>M75</td>
<td>Newtonian</td>
<td>25% water</td>
<td>1022</td>
<td>13.6</td>
<td>32.1</td>
</tr>
<tr>
<td>M50</td>
<td>Newtonian</td>
<td>60% water</td>
<td>1015</td>
<td>5.9</td>
<td>28.6</td>
</tr>
<tr>
<td>D100</td>
<td>Newtonian</td>
<td>neat</td>
<td>1045</td>
<td>17.8</td>
<td>39.7</td>
</tr>
<tr>
<td>D75</td>
<td>Newtonian</td>
<td>neat</td>
<td>1034</td>
<td>8.4</td>
<td>31.7</td>
</tr>
<tr>
<td>A100</td>
<td>non-Newtonian</td>
<td>neat</td>
<td>1040</td>
<td>1149.5</td>
<td>32.2</td>
</tr>
<tr>
<td>A50</td>
<td>non-Newtonian</td>
<td>50% water</td>
<td>1020</td>
<td>661.5</td>
<td>31.7</td>
</tr>
<tr>
<td>A25</td>
<td>non-Newtonian</td>
<td>75% water</td>
<td>1010</td>
<td>168.5</td>
<td>29.8</td>
</tr>
</tbody>
</table>
Figure 59. Variation of viscosity with shear rate for some of the fluids tested in ref. 25.

The linear stability code developed in ref. 25 produces neutral stability curves in Re – α plane (figure 60a), amplification rates (σ) and phase velocity variations as a function of wave number (figure 60 b) at fixed Re-number based on liquid depth (Ud/u). When no artificial excitation device is used, the waves observed will be the ones corresponding critical conditions. At the critical Re-number (fig. 60 a) one wave is neutrally stable and all others are damped.

The linear instability code needs as inputs viscosity ratio, density ratio, the fluid film thickness, surface tension coefficient and the skin friction coefficient $C_f$. Quoting ref. 25: “The choice of $C_f$ is not straight forward task (sic!) and in general detailed boundary layer measurements need to be done.” Özgen et al has chosen a fixed value of $C_f = 0.0025$ which is slightly below the value given by Schlichting formula:

$$C_f = 0.0576 \, \text{Re}^{-1/5}_x$$

According to Özgen et al. the mode computed corresponded the interfacial mode as defined by Miesen and Boersma (ref. 20) as the phase velocities were always greater than the interfacial velocity. The internal mode was also captured by the code, for which the configuration was always stable and so irrelevant for the study.

The calculated results are compared with the measured ones for Newtonian (deicing) fluids in table 6 and for non-Newtonian (anti-icing) fluids in table 7.
Figure 60. Results of linear instability code for M100 fluid with d = 2.0 mm

Table 6. Calculated and measured wave characteristics for Newtonian fluids.
Table 7. Calculated and measured wave characteristics of non-Newtonian fluids.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$d_f^I$ (mm)</th>
<th>$d_f^II$ (mm)</th>
<th>$U_{cr.I}$ (m/s)</th>
<th>$U_{cr.II}$ (m/s)</th>
<th>$U_{cr}$ (m/s)</th>
<th>$R_c$</th>
<th>$R_{cr}$</th>
<th>$\lambda^+(e)$ (mm)</th>
<th>$\lambda^+(u)$ (mm)</th>
<th>$c^+(e)$ (m/s)</th>
<th>$c^+(u)$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A100</td>
<td>2.4</td>
<td>2.2</td>
<td>11.4</td>
<td>11.0</td>
<td>0.084</td>
<td>2.8E-6</td>
<td>1661</td>
<td>27.5</td>
<td>28.4</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>A100</td>
<td>1.5</td>
<td>1.5</td>
<td>11.9</td>
<td>10.7</td>
<td>0.052</td>
<td>9.6E-6</td>
<td>1101</td>
<td>23.6</td>
<td>31.2</td>
<td>1.1</td>
<td>0.4</td>
</tr>
<tr>
<td>A100</td>
<td>0.7</td>
<td>0.7</td>
<td>12.9</td>
<td>9.8</td>
<td>0.017</td>
<td>8.9E-8</td>
<td>468</td>
<td>22.4</td>
<td>38.9</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>A50</td>
<td>1.9</td>
<td>1.9</td>
<td>12.0</td>
<td>10.8</td>
<td>0.132</td>
<td>3.2E-6</td>
<td>1409</td>
<td>30.2</td>
<td>29.3</td>
<td>2.3</td>
<td>1.2</td>
</tr>
<tr>
<td>A50</td>
<td>1.4</td>
<td>1.4</td>
<td>12.0</td>
<td>10.6</td>
<td>0.088</td>
<td>1.3E-6</td>
<td>1015</td>
<td>26.3</td>
<td>30.0</td>
<td>1.6</td>
<td>0.6</td>
</tr>
<tr>
<td>A25</td>
<td>0.8</td>
<td>0.8</td>
<td>12.3</td>
<td>9.8</td>
<td>0.037</td>
<td>2.0E-7</td>
<td>536</td>
<td>28.8</td>
<td>34.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>A25</td>
<td>1.9</td>
<td>2.0</td>
<td>11.5</td>
<td>10.8</td>
<td>1.9</td>
<td>1.3E-3</td>
<td>1475</td>
<td>21.7</td>
<td>30.2</td>
<td>16.2</td>
<td>16.2</td>
</tr>
<tr>
<td>A25</td>
<td>1.5</td>
<td>1.7</td>
<td>11.9</td>
<td>10.7</td>
<td>1.8</td>
<td>1.0E-3</td>
<td>1242</td>
<td>20.4</td>
<td>29.4</td>
<td>12.6</td>
<td>12.2</td>
</tr>
<tr>
<td>A25</td>
<td>0.9</td>
<td>1.0</td>
<td>11.9</td>
<td>10.1</td>
<td>0.9</td>
<td>2.4E-4</td>
<td>694</td>
<td>20.5</td>
<td>34.0</td>
<td>1.3</td>
<td>2.7</td>
</tr>
</tbody>
</table>

The columns in tables 6 and 7 are as follows:

<table>
<thead>
<tr>
<th>Column No</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fluid type and dilution – see table 5</td>
</tr>
<tr>
<td>2</td>
<td>Fluid film depth before the test</td>
</tr>
<tr>
<td>3</td>
<td>Fluid film depth during the test</td>
</tr>
<tr>
<td>4</td>
<td>Critical air velocity by experiments – the first wave appears</td>
</tr>
<tr>
<td>5</td>
<td>Critical air velocity as calculated from critical Re-number (see figure 60 a)</td>
</tr>
<tr>
<td>6</td>
<td>Interface velocity calculated by the instability code</td>
</tr>
<tr>
<td>7</td>
<td>Re-number of the fluid calculated by the instability code ($= \rho_{fluid}U_o^{2-n}d/k^n$)</td>
</tr>
<tr>
<td></td>
<td>where $n=1$ for Newtonian fluids and $k^1 = \mu$. See also comments on ref. 19.</td>
</tr>
<tr>
<td>8</td>
<td>Critical Re-number calculated by the instability code</td>
</tr>
<tr>
<td>9</td>
<td>Measured wave length</td>
</tr>
<tr>
<td>10</td>
<td>Wave length calculated by the instability code</td>
</tr>
<tr>
<td>11</td>
<td>Measured wave speed (phase speed)</td>
</tr>
<tr>
<td>12</td>
<td>Wave speed calculated by the instability code</td>
</tr>
</tbody>
</table>

There seems to quite good agreement with the measured and calculated values of critical air velocities in the Newtonian fluids. The same applies to non-Newtonian fluids however compared to references 4 and 7 the values are approximately 50% lower. The comparison between calculated and measured wave speeds are as well in good agreement. The non-Newtonian wave speeds are however extremely low. The first waves measured in ref. 4 for Type IV fluid were two orders of magnitude faster than the ones in ref. 25. Özgen et al comments this as follows: “Although they appear at roughly the same velocities as for Newtonian fluids enough time should be given to non-Newtonian waves to develop. Another problem is the low wave speeds which often is of the same order of magnitude as the experimental uncertainty. This is primarily due to high viscosity values of non-Newtonian fluids. After the appearance of quasi two dimensional waves if the
tunnel speed is further increased triangular waves start appearing, which are much faster than two dimensional waves. They originate at a point and develop both in stream wise and span wise directions at comparable rates. These waves sweep through the test section at very short time and enhance transportation of fluid towards the exit. The points of origin of these waves are probably points of very high shear resulting from turbulent structures within the air flow.”

Özgen et al. claims that non-Newtonian waves are not as two dimensional as Newtonian ones. This might be true for their measurements but to compare the waves in ref. 4 and ref. 25 in figures 61 a) and b) it is obvious that the two orders of magnitude faster waves in figure 61 b) are quite two dimensional.

Figure 61. Wave images from experiments in ref. 25 (a) and ref. 4 (b)

One shortcoming in Özgen et al measurements is the lack of information about measured interface velocity or any other flow rate information as well. There is a description of fluid thickness in test section during a test in figure 62 which reveals that the authors of ref. 25 obviously are interested only on the first 30 seconds of the experiments when the wave front of the fluid is still within the test section and there is no actual flow. Something essential in the process is however left out of the focus if the flow off is neglected and the interface velocity is said to be almost nil.

One important observation in ref. 25 is that wave speeds are decreasing with decreasing fluid thickness and this is becoming sharper as the fluid thickness falls typically below 1 mm. A similar finding was done in ref. 4.
An obvious exception from the mainstream of the literature on de/anti-icing fluid behavior on the wing is Dart’s paper (ref. 27) on fluid flow off from wings. Dart derives a numerical method to calculate the fluid flow off based merely on aerodynamic shear force. He is the first one to give clear figures on fluid flow off volumes in time from a 2 dimensional wing section. However the method is very simplified neglecting the waves and possible pressure forces on the waves totally. According to Dart: “The aerodynamic shear force is the most significant force that is involved in the transportation of the deicing fluid during take-off and is the only force that is currently considered by the model. This is thought to be an acceptable simplification since other forces, such as gravitational forces and pressure forces are judged to be insignificant when compared to the aerodynamic shear force.” Dart is most obviously unaware about theories of Rothmayer and Tsao (ref. 23).

What is positive in darts method is that the reader has no problems following the derivation of his method. For this reason the derivation is presented unabbreviated in the following. figure 63 illustrates the nomenclature of the method.

The numerical method is based on Newton’s equation for viscosity:

\[ \tau = \mu \frac{du}{dy} \]

Where \( \mu \) is the dynamic viscosity of the fluid, \( \tau \) is the aerodynamic shear force and \( \frac{du}{dy} \) is the velocity gradient through the fluid film. It is assumed that the fluid film is thin enough for the velocity gradient to be constant which leads after integration of the previous equation to:

\[ u = \frac{\tau y}{\mu} \]
The mass flow rate per unit span at any location on the airfoil chord can be obtained by integration of the velocity:

\[ m = \rho_f \int u \, dy \]

where \( \rho_f \) is the density of the fluid and \( h \) the thickness of the fluid film (fig. 63). Substituting the velocity into the integral gives:

\[ m = \rho_f \int \frac{\tau v}{\mu} \, dy \]

\[ = \rho_f \left[ \frac{\tau y^2}{2\mu} \right]_0^h \]

\[ = \rho_f \frac{\tau h^3}{2\mu} \]

The fluid transportation is computed by discretizing the upper surface of the wing and by considering the mass flow in and out of the control volumes formed by the discretization (figure 64). Using the previous equation and this finite difference approach, the fluid mass flow rate parameters illustrated in fig. 64 can be expressed as follows:

\[ m_{in} = \frac{\rho_f \tau_{i-1} h_{i-1}^3}{2\mu} \quad m_{out} = \frac{\rho_f \tau_i h_i^3}{2\mu} \]

The net increase of the fluid mass at node \( i \) is:

\[ m_{net} = m_{in} - m_{out} \]

Substituting the mass flows in and out of the control volume we get:

\[ m_{net} = \rho_f \left( \frac{\tau_{i-1} h_{i-1}^3 - \tau_i h_i^3}{2\mu} \right) \]
The rate of change of fluid thickness is:

\[ \dot{h}_i = \frac{\dot{m}_{net}}{\Delta x} \]

in which:

\[ \dot{m}_{net} = \frac{\dot{m}_{in}}{\rho_f} \Delta x = \frac{x_{r1} - x_{r-1}}{2} \]

So we get

\[ \dot{h}_i = \frac{2 \rho_f \left( \tau_{r1} h_{r1}^2 - \tau_i h_i^2 \right)}{\rho_f (x_{r1} - x_{r-1})} \]

\[ \Rightarrow \quad \dot{h}_i = \frac{\tau_{r1} h_{r1}^2 - \tau_i h_i^2}{\mu (x_{r1} - x_{r-1})} \]

The height of the fluid thickness after time step \( \Delta t \) is obtained from the following:

\[ h_i = h_{i-1} + \dot{h}_i \Delta t \]

For a known shear stress distribution and an initial film thickness the fluid height as a function of time can be solved now numerically. According to ref. 27, the given viscosity may be either a constant value (Newtonian) or may be defined in terms of the local shear stress to represent a non-Newtonian fluid. However the local shear stress is not the parameter to define the power law viscosity but the local shear rate. There should be an iterative process to define a relevant local non-Newtonian viscosity. However this is not addressed in ref. 27.

The aerodynamic shear stress as a function of time is an input for the method. These are obtained in Dart’s study from an inflight ice accretion prediction code.
(ref. 28) that uses a panel method and integral boundary layer method to calculate skin friction coefficients.

The method is validated (though Dart uses term verification) with test cases of flat plate and two different airfoils. In flat plate case he illustrates the fluid flow off unfortunately on a 0.5 m flat plate with a constant air velocity of 50 m/s. This kind of a case does not give anything to compare with. If there had been an acceleration from 0 to 60 m/s velocity and a 1.5 m flat plate the validation would have been possible for all fluids tested in AMIL. Figure 65 illustrates the flat plate case which is impossible to comment on due to poor parameter selection.

![Flat Plate Case](image)

Figure 65. Predicted behavior of deicing fluid for fixed $C_f$ and velocity of 50 m/s. Red lines: time steps from 0.2 to 2.6 s, blue lines from 2.8 s to 4.6 s and yellow lines from 5.6 s to 9.6 s.

The viscosity selected to the case of fig. 65 is 150 mPa s and the fixed $C_f = 0.02$ which is once again a completely unrealistic value.

The second validation case is familiar as it is the Boeing airfoil in ref. 3 – or its one element simplified version (figure 66). This time the calculations have been done for a take-off simulation with an acceleration to velocity of 69.5 m/s within 30 second time.

![Boeing Case](image)

Figure 66. Boeing 737 – 200 AD profile (flaps $5^\circ$) and its one element equivalent.

The Boeing wing profile results are presented in figure 67. It is a full scale case with chord of 2.54 m. ref 3 does not include any tests of this airfoil with Type I fluids however there is one relevant test in ref. 29 with a 2D model of the same profile with a chord of 0.46 which is almost twice the model in ref. 3. The Type I fluid depth measured during an acceleration is presented in figure 68. Note the early rotation.
Figure 67. Fluid flow-off calculated with the method of ref. 27 for Boeing 737-200 AD. Blue lines: time from 0.5 s to 6.5 s, green lines from 7 to 11.5 seconds, yellow lines from 14 to 24 seconds, violet line at 30 seconds after start of acceleration.

Though Dart does not compare his results to anything it is now possible to compare his calculations to experiments. The initial fluid thicknesses between the calculation and experiments naturally differ quite a lot – being 2 mm in the case of fig 67 and 0.75 mm in fig. 68. Note the error in the text of fig. 68. As the initial thickness is 0.75 the mean thickness after 10 seconds should probably be either 0.566 or 0.666 instead of 0.166 mm. Otherwise the fluid on the wing is increasing not decreasing between 10 and 24 seconds.

As the initial thicknesses of the two differs the relative thicknesses has to be compared. Though Dart claims the method of his gives realistic progress of fluid flow one might disagree too according to these figures. On the other hand the flow off process may be quite different with fluid films having this different initial thicknesses.

There is no exact data in ref. 27 on how the friction coefficient values have been determined however there is one figure (fig. 69) showing the shear stress distribution for two wing profiles (Boeing and Airbus). Calculating the corresponding friction coefficient from this figure with airspeed of 69.5 m/s we get a maximum value of $C_f = 0.08$ and an average value of 0.025. Both values are very high at least if we compare the values of Özgen et al in ref. 25. This gives an impression that as the waves and pressure forces have been neglected this has been compensated with friction coefficients unnaturally high.

The method of Dart might be somewhat inaccurate nevertheless it is the first to give flow off rates of same order of magnitude as measured ones (!).
Figure 68. Measured deicing film thickness for Boeing 737 200 AD wing section (flaps 5°) from ref. 29. a) 10 s airspeed 24.2 m/s  b) 24 s airspeed 61.2 m/s c) after rotation.
4 General comments on the literature reviewed

There is a lot of reports considered as “fundamental” in the literature of the considered branch that was left over from this review. The reason is clear – although these reports are most probably academically valuable their added value in practical engineering modelling purposes are next to nothing. However in case the reader wants dig deeper into the world of instability theories of relevant flows considering water or deicing fluids references 30, 31, 32 and 33 are the first “fundamental” studies to start with.

After reading all the 33 references on the list of this review one particular issue come up as first. The majority of articles even in the most respected series (Journal of Fluid Mechanics, Physics of Fluids, Journal of Engineering Mathematics etc) are below all standards. Nomenclature is a rarity, found only in AIAA journals, a lot of variables, terms and concepts fall out of the sky without explanations. The worst papers are like notes for authors themselves without a single thought for the reader. All kind of errors are more a standard than a deviation. Correction of these are left to the reader. It is difficult to avoid the idea of a “club” of researchers – once you get in anything goes. Once you are a reviewer you don’t have to behave at all.

What it comes to the utility of most of the papers referred in this review the added value of them is infinitesimal. A good example is the approach to the airflow boundary layer. Overwhelming majority is considering Blasius flow and Tollmien Schlichting modes although 100 % of the practical airliner wing cases are groveling in a fully developed turbulent boundary layer – even partly separated in worst cases. The second issue is the time dependence. Excluding Darts perhaps undetailed nevertheless practical approach there is not a single study trying to address the time dependent situation of fluid flow off process. The same applies to the volume flow. Excluding Rothmayer and Tsao’s analysis and once again Dart’s engineering analysis there is nothing on fluid flow off. The wave lengths and amplification factors calculated over and over again does not give any clue about how fast the fluid is going to disappear from the wing.

One particular oddity are the unrealistic friction coefficients appearing too often. This strain applies to the paper of Dart too. If friction coefficients of 0.02 are a general phenomenon in the fluid dynamic literature it should be studied more closely as a general plausibility.
5 References


22. Jameson, A. Time dependent calculations using multigrid, with applications to unsteady flow past airfoils and wings. AIAA 91-1596.


